HWK §5.5

Thursday, March 18, 2021 10:21 AM

#7 $\{u_1, u_2, u_3\}$ ⊥-normal

$x = c_1 u_1 + c_2 u_2 + c_3 u_3$ with $\|x\| = 5$, $\langle x, u_1\rangle = 4$, $x \perp u_2$

Find possible values of c_1, c_2, c_3.

$c_1 = \langle x, u_1\rangle = 4$, $c_2 = \langle x, u_2\rangle = 0$

$5^2 = \|x\|^2 = c_1^2 + c_2^2 + c_3^2 = 4^2 + c_3^2 \Rightarrow c_3 = \pm 3$.

#8 $\{\sin x, \cos x\}$ ⊥-normal. $f(x) = 3\cos x + 2\sin x$

$g(x) = \cos x - \sin x$

$\langle f, g\rangle = \langle 3\cos x + 2\sin x, \cos x - \sin x\rangle$

$= 3\langle \cos x, \cos x\rangle - 3\langle \cos x, \sin x\rangle + 2\langle \sin x, \cos x\rangle$

$- 2\langle \sin x, \sin x\rangle$

$= 3 * 1 - 3 * 0 + 2 * 0 - 2 * 1 = 1$.

#9 $\{\frac{1}{\sqrt{2}}, \cos x, \cos 2x, \cos 3x, \cos 4\}$ ⊥-normal set

$\langle f, g\rangle = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x) g(x)\,dx$.

$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$, $\sin^4 x = \frac{1}{4}(1 - \cos 2x)^2 = \frac{3}{4} - \frac{1}{2}\cos 2x + \frac{1}{4}\cos 4x$

a) $\int_{-\pi}^{\pi} \sin^4 x \cos x\,dx = \int_{-\pi}^{\pi} \cos x\left(\frac{3}{4} - \frac{1}{2}\cos 2x + \frac{1}{4}\cos 4x\right)dx = 0$

$-\pi$ $-\pi$

b) $$\int_{-\pi}^{\pi} \sin^4 x \cos 2x \, dx = \int_{-\pi}^{\pi} \cos 2x \left(\frac{3}{4} - \frac{1}{2}\cos 2x + \frac{1}{4}\cos 4x\right) dx$$

$$= -\frac{1}{2}\int_{-\pi}^{\pi} \cos^2 2x \, dx = -\frac{\pi}{2}$$

c) $$\int_{-\pi}^{\pi} \sin^4 x \cos 3x \, dx = \int_{-\pi}^{\pi} \cos 3x \left(\frac{3}{4} - \frac{1}{2}\cos 2x + \frac{1}{4}\cos 4x\right) dx = 0$$

d) $$\int_{-\pi}^{\pi} \sin^4 x \cos 4x \, dx = \int_{-\pi}^{\pi} \cos 4x \left(\frac{3}{4} - \frac{1}{2}\cos 2x + \frac{1}{4}\cos 4x\right) dx$$

$$= \frac{1}{4}\int_{-\pi}^{\pi} \cos^2 4x \, dx = \frac{\pi}{4}.$$